INTERNATIONAL JOURNAL FOR NUMERICAL AND ANALYTICAL METHODS IN GEOMECHANICS, VOL. 21, 619–633 (1997)

A MIXED SOLUTION FOR TWO-DIMENSIONAL UNSTEADY FLOW IN FRACTURED POROUS MEDIA

C. FIDELIBUS,[1] G. BARLA[2,]* AND M. CRAVERO[3]

[1] *CNR, Centro Studi sulle Risorse Idriche e la Salvaguardia del Territorio, Politecnico di Bari, Via E. Orabona 4, 70125 BARI, Italy*
[2] *Structural Engineering Department, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*
[3] *CNR, Centro Studi per la Fisica delle Rocce e le Geotecnologie, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*

SUMMARY

A mixed finite element–boundary element solution for the analysis of two-dimensional flow in porous media composed of rock blocks and discrete fractures is described. The rock blocks are modelled implicitly by using boundary elements whereas finite elements are adopted to model the discrete fractures. The computational procedure has been implemented in a hybrid code which has been validated first by comparing the numerical results with the closed-form solution for flow in a porous aquifer intercepted by a vertical fracture only. Then, a more complex problem has been solved where a pervious, homogeneous and isotropic matrix containing a net of fractures is considered. The results obtained are shown to describe satisfactorily the main features of the flow problem under study. © 1997 by John Wiley & Sons, Ltd.

Int. J. Numer. Anal. Meth. Geomech., Vol. 21, 619–633 (1997)
(No. of Figures: 11 No. of Tables: 1 No. of Refs: 10)

Key words: unsteady flow; fractured porous media; boundary elements; finite elements; hybrid code; validation of numerical solution

1. INTRODUCTION

The adoption of appropriate characterization methods based on realistic assumptions for the quantification of the hydraulic properties is essential in engineering applications dealing with the groundwater flow in geological media. With this in mind, a computational method is described in the present paper which allows one to represent the medium as composed of porous blocks separated by discrete fractures.

The typical conditions where this method is applicable are referred to rock masses where either multiple fractures are present, together with a permeable matrix, or the rock blocks are intersected by a large number of fractures so as to allow for the evaluation of equivalent hydraulic properties. In the latter case an equivalent continuum is therefore associated to fractures which should be accounted for independently.

The solution of the flow problem in transient state is obtained by using a numerical model where the Boundary Element Method (BEM) is adopted for the continuum blocks and the Finite

*Correspondence to: G. Barla, Structural Engineering Department, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

CCC 0363–9061/97/090619–15$17.50

Received 28 May 1996
Revised 22 January 1997

Element Method (FEM) is used for the discrete fractures. The model may provide the basis for the solution of mass transport problems of the particle tracking type.

2. MATHEMATICAL APPROACH

As shown in Figure 1, let one define the flow domain Ω with the contour Γ, where as well defined are: *nr* fracture branches (R_k), *nb* subdomains (Ω_l), each one coinciding with the porous blocks (B_l); also defined are *ni* intersection of fractures (I_m), comprising the intersection with the contour domain.

The solution of the parabolic equations governing the hydraulic flow within the domain are dependent upon the following boundary conditions on the contour Γ:

$$\begin{aligned} h(P,t) &= \bar{h}, \quad P \in \Gamma_A, \quad t > t_0 \\ q_n(P,t) &= \bar{q}_n, \quad P \in \Gamma_B, \quad t > t_0 \\ Q(P,t) &= \bar{Q}, \quad P \in (\Gamma \cap R_k), \quad t > t_0 \end{aligned} \tag{1}$$

with the following condition at time t_0:

$$h(P, t_0) = h_0, \quad P \in \Omega \tag{2}$$

where Γ_A and Γ_B are, respectively, the contours where the Dirichlet's and Neumann's conditions apply, h is the piezometric head, q_n the specific discharge component (positive according to the outward normal), and Q is the volumetric flow rate out of the given branch.

When describing a porous rock mass with discrete fractures, the flow conditions within the same fractures are written separately from those pertaining to the porous blocks. Given that the

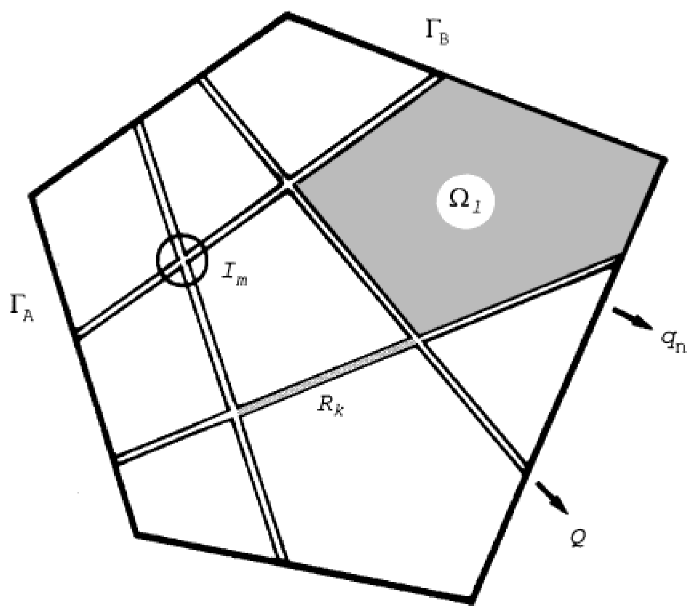

Figure 1. Flow domain comprising porous blocks and fractures

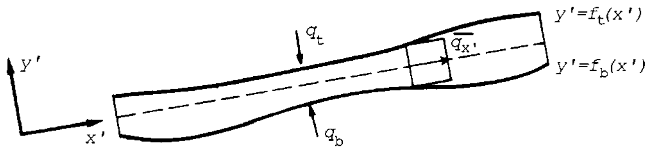

Figure 2. Local co-ordinate system for a given fracture

volume of the discrete fractures is a reduced portion of that comprising the porous blocks, the analytical scheme to be used consists in defining for the fractures a one-dimensional flow equation.

2.1. Continuity equation within fractures

Let one consider a fracture branch R_k and the associated local coordinate system x', y'. The x'-axis is directed along the segment containing the fracture as shown in Figure 2. The geometry of the fracture walls is such that the flow rate obeys the 3rd power law, under the assumption of laminar flow. This condition implies that a fracture can be represented by its axis, without any significant change in direction, whereas the variation of the aperture from the mean value can be described by the geometry of the walls, which are defined according to the functions f_t and f_b as given in Figure 2.

The continuity equation for the elementary volume dx', dy' within a given fracture is given as follows:[1]

$$-\operatorname{div}\mathbf{q} = \phi_F \beta \frac{\partial p}{\partial t} + \frac{1}{e}\frac{\partial}{\partial t}(u_t - u_b) \tag{3}$$

where $q(x', y')$ is the specific flow rate, β is the fluid compressibility, ϕ_F the porosity of the fracture filling, if present, $p(x', y')$ the pressure, $e(x') = f_t(x') - f_b(x')$ the fracture mechanical aperture, $u_t(x')$ and $u_b(x')$ the displacement components of the walls along y'.

The volumetric strain in equation (3) is assumed to coincide with the component in the y'-direction; it is also assumed that there is no coupling among the displacement components along the two axes. Therefore, by integrating equation (3) along y', one obtains

$$\int_{y'=f_b(x')}^{y'=f_t(x')} -\operatorname{div}\mathbf{q}\,dy' = \int_{y'=f_b(x')}^{y'=f_t(x')} \left[\phi_F \beta \frac{\partial p}{\partial t} + \frac{1}{e}\frac{\partial (u_t - u_b)}{\partial t}\right] dy' \tag{4}$$

Then, the first integral of the above equation (4) allows one to write:

$$\int_{y'=f_b(x')}^{y'=f_t(x')} -\operatorname{div}\mathbf{q}\,dy' = \int_{y'=f_b(x')}^{y'=f_t(x')} -\frac{\partial q_{x'}}{\partial x'}\,dy' - q_{y'}|_{y'=f_t(x')} + q_{y'}|_{y'=f_b(x')} \tag{5}$$

where $q_{x'}$ and $q_{y'}$ are the specific flow rate components.

If the mean specific flow rate is introduced as

$$\bar{q}_{x'} = \frac{1}{e}\int_{y'=f_b(x')}^{y'=f_t(x')} q_{x'}\,dy' \tag{6}$$

and the $q_{y'}$ components in equation (5) are chosen to coincide with q_t and q_b according to the normal to the fracture walls, the integral (5) can be written as

$$\int_{y'=f_b(x')}^{y'=f_t(x')} -\operatorname{div}\mathbf{q}\,\mathrm{d}y' = -\frac{\mathrm{d}(e\bar{q}_{x'})}{\mathrm{d}x'} + q_t + q_b \tag{7}$$

The relation between the specific flow rate and the hydraulic gradient can be written as[2]

$$q_{x'} = -\frac{e^2}{12\mu}\frac{\mathrm{d}h}{\mathrm{d}x'} \tag{8}$$

given that the flow along the fracture is along the x'-axis and is of the laminar type.

Substituting the constitutive equation (8) into (6), the flow rate $Q(x')$ for the cross section of the fracture is obtained:

$$\begin{aligned} Q(x') = e\bar{q}_{x'} &= -\frac{e^2}{12\mu}\int_{-e/2}^{e/2}\frac{\mathrm{d}h}{\mathrm{d}x'}\mathrm{d}y' = -\frac{e^2}{12\mu}\frac{\mathrm{d}}{\mathrm{d}x'}\left[\int_{-e/2}^{e/2} h\,\mathrm{d}y'\right] = -\frac{e^2}{12\mu}\frac{\mathrm{d}(e\bar{h})}{\mathrm{d}x'} \\ &= -\frac{e^2}{12\mu}\left(e\frac{\mathrm{d}\bar{h}}{\mathrm{d}x'} + \bar{h}\frac{\mathrm{d}e}{\mathrm{d}x'}\right) \end{aligned} \tag{9}$$

which, under the assumption that the fracture aperture gradient is negligible, can be represented more conveniently as

$$Q(x') = -\frac{e^3}{12\mu}\frac{\mathrm{d}\bar{h}}{\mathrm{d}x'} = -K_F\frac{\mathrm{d}\bar{h}}{\mathrm{d}x'} \tag{10}$$

where $K_F(x')$ is the fracture transmissivity.

Then equation (5) can finally be written as

$$\int_{y'=f_b(x')}^{y'=f_t(x')} -\operatorname{div}\mathbf{q}\,\mathrm{d}y' = \frac{\mathrm{d}}{\mathrm{d}x'}\left(K_F\frac{\mathrm{d}h}{\mathrm{d}x'}\right) + q_t + q_b \tag{11}$$

It is noted that the above equation (11) when set equal to 0 coincides with that for the steady-state condition.[3] This is a one-dimensional flow equation and is well applicable given that the volume of the fracture is sufficiently small compared with the volume of the rock mass which is intersected. In this way the flow within the fracture opening is referred to its axis and the walls pertaining to the two blocks are made to coincide with it.

In order to be able to evaluate the second integral in equation (4), the following assumptions need be introduced:

- the density of the fluid, which is nearly incompressible, is invariant with respect to its position; then, according to the definition given for the piezometric head:

  $$h = p/\rho g + z$$

  where z is the geometric height and ρ the fluid density, one may write:

  $$\frac{\partial p}{\partial t} = \rho g\frac{\partial h}{\partial t} \tag{12}$$

- the changes in effective stress are to coincide with the pressure changes, for a constant total stress; then one may write along the y'-axis:

$$\frac{1}{e}\frac{\partial}{\partial t}(u_{\mathrm{t}}-u_{\mathrm{b}})=C_{\mathrm{F}}\frac{\partial p}{\partial t}=C_{\mathrm{F}}\rho g\frac{\partial h}{\partial t} \tag{13}$$

where C_{F} is the specific compressibility of the fracture.
Finally, one may write the following equation:

$$\int_{y'=f_{\mathrm{b}}(x')}^{y'=f_{\mathrm{t}}(x')}\left[\phi_{\mathrm{F}}\beta\frac{\partial p}{\partial t}+\frac{1}{e}\frac{\partial(u_{\mathrm{t}}-u_{\mathrm{b}})}{\partial t}\right]\mathrm{d}y'=\int_{y'=f_{\mathrm{b}}(x')}^{y'=f_{\mathrm{t}}(x')}\rho g(\phi_{\mathrm{F}}\beta+C_{\mathrm{F}})\frac{\partial h}{\partial t}\mathrm{d}y'$$

$$=e\rho g(\phi_{\mathrm{F}}\beta+C_{\mathrm{F}})\frac{\partial h}{\partial t}=S_{\mathrm{F}}\frac{\partial h}{\partial t} \tag{14}$$

where S_{F} is the specific storage, i.e. the fluid volume which is released (or stored) within the fracture for a unit change of the hydraulic head and for a unit fracture area.[4] The one-dimensional form of equation (3) results to be given as

$$\frac{\partial}{\partial x'}\left(K_{\mathrm{F}}\frac{\partial h}{\partial x'}\right)+q_{\mathrm{t}}+q_{\mathrm{b}}=S_{\mathrm{F}}\frac{\partial h}{\partial t} \tag{15}$$

where, as a first approximation, the transmissivity K_{F} is considered to be marginally influenced by a change in fracture thickness during the unsteady flow.

2.2. *Continuity equation for the porous blocks*

The parabolic equation governing the diffusion within the domain Ω_1 of a given porous block B_1, which can be described in a x, y co-ordinate system taken with the axes coinciding with the principal directions of the hydraulic conductivity is[5]

$$K_{\mathrm{B}x}\frac{\partial^2 h}{\partial x^2}+K_{\mathrm{B}y}\frac{\partial^2 h}{\partial y^2}+Q_{\mathrm{s}}(W)=S_{\mathrm{B}}\frac{\partial h}{\partial t} \tag{16}$$

where $K_{\mathrm{B}x}$ and $K_{\mathrm{B}y}$ are the principal values of the conductivity, S_{B} is the specific storage capacity of the block and $Q_{\mathrm{s}}(W)$ is a constant point source.

2.3. *Continuity equation at the intersections*

The continuity equation at the intersection can be written as

$$\sum_{j=1}^{nF(i)}(\mathbf{x}'_j\cdot\mathbf{x}_j^{\mathrm{IN}})Q_j(t)+Q^*=0 \tag{17}$$

where $nF(i)$ is the number of fracture branches intersecting at i, Q_j the flow rate characterizing a fracture branch j, $\mathbf{x}'_j$ is the unit vector arbitrarily assigned to the same branch, $\mathbf{x}_j^{\mathrm{IN}}$ the unit vector parallel to the fracture branch but entering the node, Q^* the fluid entering (positive) or released (negative) in the unit time and for a unit thickness. It is assumed that no storage capacity exists at the intersection.

3. NUMERICAL SOLUTION SCHEME

Different numerical approaches are available for the numerical solution of flow in a porous medium intersected by a number of discrete fractures; Huyakorn *et al.*[6] apply the Galerkin method to the continuity equations for both the fractures and the blocks and discretize the system by using finite elements; Baca *et al.*[7] adopt a variational principle and the superposition method, by using again the finite element method.

The main difficulty associated with these approaches is on one side the discretization of the entire domain, including the porous blocks and the fractures, and on the other side the need to increase the degree of discretization where high gradients are expected to occur. This approach is quite complex and affected by a certain degree of uncertainty on the type of discretization used, mainly if no adaptive techniques are adopted. Additionally, the discontinuity in the flow field, which is associated with the use of standard finite elements where the continuity of the derivatives is neglected, make it difficult to use the output in mass transport models.

With the purpose to avoid these difficulties, the Green's identities have been used to write the equations along the boundaries of each block. Given that these equations are written in explicit form as function of both the piezometric heads and the flow rates within the fractures, the discretization has been limited to the fracture branches and to the domain contour. This solution scheme, already introduced for the steady-state condition[3], has been extended to the unsteady state. Then, the solution is obtained by applying BEM to equation (16) for the porous blocks and FEM (weighted residuals according to Galerkin) to equation (15) for the discrete fractures.

It is important to underline the fact that the discrete model adopted works with a variable number of mixed degrees of freedom (piezometric head h, normal boundary flux q_{n}) according to the position of the node: if this pertains to either the block or the fracture, the number is 2 (piezometric head and normal flux); if this is located on the external contour, the number is 1 (piezometric head or normal flux). As a consequence, it is essential to use approximation schemes which are independent for the two variables.

Each fracture branch and each external contour are subdivided in three-node straight elements, where a quadratic shape function is adopted for the piezometric heads. For the fracture branch R_k one writes the following approximation:

$$h(x', t) = \sum_{j=1}^{n(k)} N_j(x') h_j(t) \tag{18}$$

where $h_j(t)$ are the values at the nodes and $N_j(x')$ the functions resulting from the assemblage of the shape functions defined for each element.

Both the fluxes q_{t} and q_{b} between a fracture branch and the adjacent blocks and the fluxes q_{n} through the contour Γ are taken as constants and are associated to the centroid of each element.

By applying the weighted residual expression to equations (15) written for the R_k branch and by integrating, one obtains

$$K_{\mathrm{F}} \frac{\partial h}{\partial x'} N_i \bigg|_{x'_0}^{x'_{\mathrm{f}}} - \int_{x'_0}^{x'_{\mathrm{f}}} K_{\mathrm{F}} \frac{\mathrm{d}N_i}{\mathrm{d}x'} \frac{\mathrm{d}N_j}{\mathrm{d}x'} \mathrm{d}x' h_j + \sum_{l=1}^{ne(k)} \int_{\Delta x'} N_i \, \mathrm{d}x' [q_{\mathrm{t},l}(x') + q_{\mathrm{b},l}(x')]$$
$$- \int_{x'_0}^{x'_{\mathrm{f}}} S_{\mathrm{F}} N_i N_i \, \mathrm{d}x' \frac{\mathrm{d}h_j}{\mathrm{d}t} = 0 \quad i, j = 1, n(k), \; l = 1, ne(k) \tag{19}$$

where $n(k)$ is the number of nodes along the fracture branch, $ne(k) = (n(k) - 1)/2$ the number of elements along the same branch, x'_0 and x'_f are the local co-ordinates of the outer nodes. The first term of equation (19) can be written as

$$- N_i(x'_f)Q_f + N_i(x'_0)Q_0,$$

with Q_f and Q_0 being the flow rates at the outer nodes. It is possible to write $n(k)$ differential equations of the same kind of equation (19), where there are $n(k)$ functions $h_j(t)$, $ne(k)$ functions $q_{t,l}(t)$ and $q_{b,l}(t)$, and two functions $Q_f(t)$ and $Q_0(t)$.

For the solution of the fracture branch it is necessary to find $2ne(k) + 2$ additional equations. To this purpose the integral equations for the porous blocks along the branch and the equilibrium equations at the intersections are used. In particular, the solution for the flux regarding the porous block is obtained by integrating the second Green's identity, with respect to time, as follows:[8]

$$\begin{aligned} c(U)h(U,t) &+ \frac{1}{S_B}\int_{t_0}^{t} dt' \int_{\Gamma_l} h(V,t')\left[K_{Bx}\frac{\partial h^*}{\partial x}(U,t|V,t')n_x + K_{By}\frac{\partial h^*}{\partial y}(U,t|V,t')n_y\right]d\Gamma_l \\ &+ \frac{1}{S_B}\int_{t_0}^{t} dt' \int_{\Gamma_l} h^*(U,t|V,t')q_n(V,t')\,d\Gamma_l = \int_{\Omega_1} h_0(Z)h^*(U,t|Z,t'_0)d\Omega_l \\ &+ \int_{t_0}^{t} dt' \int_{\Omega_1} Q_s(W)h^*(U,t|W,t')\,d\Omega_l \end{aligned} \tag{20}$$

where h^*, Green's function and singular solution of the following equation:

$$K_{Bx}\frac{\partial^2 h}{\partial x^2} + K_{By}\frac{\partial^2 h}{\partial y^2} - S_B\frac{\partial h}{\partial t} = \Delta(x_V - x_U)\Delta(y_V - y_U)\Delta(t' - t) \tag{21}$$

is given by

$$h^*(U,t|V,t') = \frac{S_B}{4\pi(K_{Bx}K_{By})^{1/2}(t-t')}e^{-R^2S_B/4(t-t')} \tag{22}$$

where U is the point on the contour where the unit impulse generating the field $h^*(t')$ at the time t is located, V and Z are points pertaining, respectively, to the contour and to the domain, $c(U)$ is a shape coefficient for the contour near to U, and q_n represents either the flux between the porous block and the fractures, or the flux through the external contour if the block is adjacent to it.

In the same equation R is defined as the distance between U and V, which is given by the following expression:

$$R = \left[\frac{(x_U - x_V)^2}{K_{Bx}} + \frac{(y_U - y_V)^2}{K_{By}}\right]^{1/2} \tag{23}$$

$q_n(V)$ can now be written as

$$q_n(V) = -\left[K_{Bx}\frac{\partial h}{\partial x}(V)n_x + K_{By}\frac{\partial h}{\partial y}(V)n_y\right] \tag{24}$$

where n_x and n_y are the components of the outward-drawn normal to the Γ contour in V. It is noted that:

1. when the contour of a block is formed by a fracture branch, $q_n(V)$ coincides with either $q_{t,l}$ or $q_{b,l}$;

2. when the contour is along the external boundary, $q_n(V)$ denotes the specific discharge component which is either entering or exiting along the direction normal to the domain Ω.

As far as the surface integral in equation (20), if the initial condition is in steady state (i.e. h_0 is harmonic), it can be transformed in an equivalent contour integral applying the Green's second identity.[9] In such a case there is no need to know the initial piezometric head distribution inside the porous block.

According to the discretization assumed along the branches, equation (20) can be written as follows:

$$c_i h_i + \frac{1}{2\pi\sqrt{K_x K_B y}} \int_\Gamma N_j \frac{D}{R^2} \left[\int_{t_0}^{t} h_j(t') a(t') \mathrm{e}^{-a(t')} \mathrm{d}t' \right] \mathrm{d}\Gamma$$

$$+ \frac{1}{4\pi\sqrt{K_{Bx} K_{By}}} \sum_{k=1}^{ne(l)} \int_{\Gamma_k} \left(\int_{t_0}^{t} q_{n,k}(t') \frac{\mathrm{e}^{-a(t')}}{a(t')} \mathrm{d}t' \right) \mathrm{d}\Gamma_K = F_{0,i}(t) + G_i(t),$$

$$i, j = 1, n(l),\ k = 1, ne(l) \tag{25}$$

where $n(l)$ is the number of nodes along the contour of a given porous block, $ne(l)$ the number of elements for the fracture branch, $F_{0,i}(t)$ the surface integral, $G_i(t)$ the contribution of the constant point source in W, and $a(t')$ is given by

$$a(t') = \frac{S_B R^2}{4(t - t')}$$

The term D stands for the distance of the point U from the element containing V.

In order to obtain a solution in the time domain, the time derivatives of equation (19) need be written in numerical form and the integrals of equation (25) are to be computed accordingly. Also to be assumed is the behaviour of $h_j(t)$, $q_{n,k}(t)$, $Q_f(t)$, $Q_0(t)$ versus time. If these functions are taken as constant in each time step Δt, which is always assumed to be the same, a solution $t_\Lambda = \Lambda \Delta t$ is obtained according to a fully implicit scheme and the following equations can be written:

for the fracture branches:

$$N_i(x_0')Q_0^\Lambda - N_i(x_f')Q_f^\Lambda + A_{ij}h_j^\Lambda + C_{il}(q_{t,l}^\Lambda + q_{b,l}^\Lambda) + \frac{B_{ij}}{\Delta t}(h_j^\Lambda - h_j^{\Lambda-1}) = 0$$

$$i, j = 1, n(k),\ l = 1, ne(k) \tag{26}$$

where

$$C_{il} = \int_{x_0'}^{x_f'} N_i \, \mathrm{d}x$$

for the intersections:

$$\sum_{j=1}^{nf(m)} Q_j^\Lambda = S^{\mathrm{I}} \frac{h_i^\Lambda - h_i^{\Lambda-1}}{\Delta t} \tag{27}$$

for the porous blocks:

$$c_i h_i^{\Lambda} + \frac{1}{2\pi\sqrt{K_{Bx}K_{By}}} \sum_{j=1}^{n(l)} \sum_{\lambda=1}^{\Lambda} \int_{\Gamma} N_j \frac{D}{R^2} [\exp(-a_{\lambda-1}) - \exp(-a_{\lambda})] d\Gamma h_j^{\lambda}$$

$$+ \frac{1}{4\pi\sqrt{K_{Bx}K_{By}}} \sum_{k=1}^{ne(l)} \sum_{\lambda=1}^{\Lambda} \int_{\Gamma_k} [E_1(a_{\lambda-1}) - E_1(a_{\lambda})]\, q_{n,k}^{\lambda}\, d\Gamma_k = F_{0,i}^{\Lambda} + \tag{28}$$

$$\frac{S_B Q_S(W)}{4\pi\sqrt{K_{Bx}K_{By}}} E_1\left(\frac{S_B R_i(W)}{4\Lambda\Delta t}\right)$$

with

$$a_{\lambda} = \frac{S_B R^2}{4\Delta t(\Lambda - \lambda)}$$

E_1 being the exponential integral and $R_i(W)$ the distance between the i node and the point W. Equation (28) can therefore be written in compact form as follows:

$$D_{ij}^{\Lambda} h_j^{\Lambda} + E_{ik}^{\Lambda} q_{n,k}^{\Lambda} = -\sum_{\lambda=1}^{\Lambda-1} D_{ij}^{\lambda} h_j^{\lambda} - \sum_{\lambda=1}^{\Lambda-1} E_{ik}^{\lambda} q_{n,k}^{\lambda} + F_{0,i}^{\Lambda} + G_i^{\Lambda} \tag{29}$$

The solution process requires one to compute the integrals D_{ij}^{λ} and E_{ik}^{λ}, which can be computed first and stored. The system of equations to be solved is composed of equations (26)–(28) with the unknowns $q_{n,k}^{\Lambda}$, Q_j^{Λ} and h_j^{Λ}.

4. NUMERICAL EXAMPLE

4.1. Case study 1

In order to validate the above computational procedure, the analytical solution, developed by Gringarten *et al.*[10] for the problem shown in Figure 3 has been taken as a case study. It is noted that a well is located at the center of a vertical fracture, with a transmissivity equal to infinity and zero storage capacity. The aquifer is assumed to be horizontal, with a unit thickness. The same problem has been previously solved by using a FEM.[6]

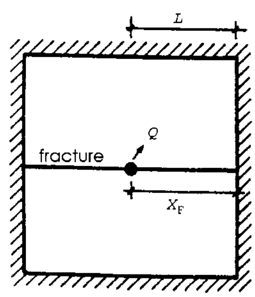

Figure 3. Case study 1: the flow domain with the assumed geometrical and hydraulic characteristics

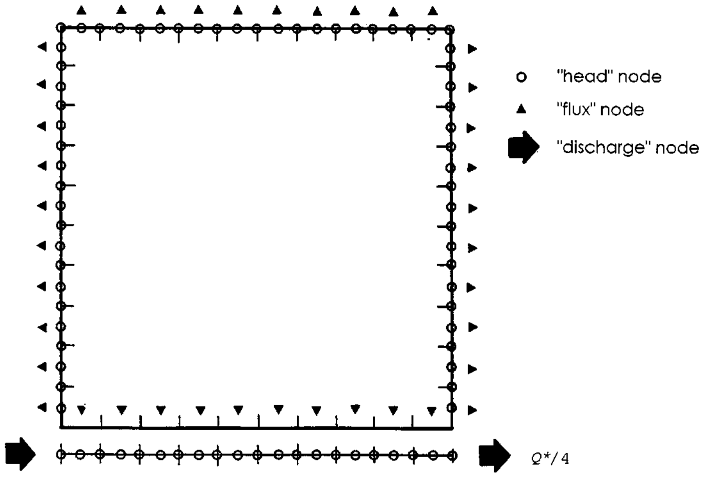

Figure 4. Domain discretization

Given the symmetry of the problem, only one-quarter of the domain has been discretized as shown in Figure 4. Two computer runs have been carried out by taking Δt equal to 4E-04 and 4E $-$ 02, respectively. The results obtained are summarized in Table I by comparing the numerical values with the analytical values computed by using the analytical solution,[10] which is given in terms of the following nondimensional parameters:

$$\frac{1}{u}=\frac{4\pi T_{\mathrm{B}} s_{\mathrm{w}}}{Q^*},\ t_{\mathrm{D}}=\frac{T_{\mathrm{B}} t}{X_{\mathrm{F}}^2 C_{\mathrm{B}}}$$

where s_w is the potential differential within the well, X_F is the half-length of the fracture, T_B and C_B are the transmissivity and the storage capacity of the aquifer, respectively, and Q^* is the volumetric flow rate which is drawn out from the well.

A similar comparison is illustrated in Figure 5 where also shown are the results obtained with FEM[6]. Excellent agreement is achieved between the numerical solution proposed above and the analytical and numerical solutions available for the flow problem taken for the purpose of validation.

4.2. Case study 2

As a second case study the plane flow problem shown in Figure 6 has been considered. The aquifer system is given a unit thickness and is formed of a pervious, homogeneous and isotropic matrix, intercepted by a net of fractures which are not all continuous. The fractures are such as to separate three matrix blocks and create a preferential flow path toward one side of the system. A constant flow rate Q_s is injected in the middle of a square region, which is characterized by zero piezometric head on the four sides. The hydraulic parameters assumed for both the matrix and the fractures are shown in the same Figure 6.

The system has been discretized along the fracture branches which separate one block from the neighbouring one, by introducing nodal points with the piezometric head unknown and nodal

Table I. Comparison with the analytical solution (*starting point for the simulation with $\Delta t = 4E\text{-}02$)

t_D	$1/u$(analyt.)	$1/u$(numer.)
4·00E − 04	0·0708	0·0696
8·00E − 04	0·1002	0·1006
1·20E − 03	0·1228	0·1232
1·60E − 03	0·1418	0·1422
2·00E − 03	0·1586	0·1590
2·40E − 03	0·1736	0·1741
3·20E − 03	0·2006	0·2010
4·00E − 03	0·2242	0·2246
8·00E − 03	0·3170	0·3175
1·20E − 02	0·3884	0·3888
1·60E − 02	0·4484	0·4488
2·00E − 02	0·5014	0·5017
2·40E − 02	0·5492	0·5496
3·20E − 02	0·6342	0·6345
4·00E − 02	0·7090	(*)0·7093
8·00E − 02	1·0026	1·0430
1·20E − 01	1·2280	1·2725
1·60E − 01	1·4184	1·4665
2·00E − 01	1·5870	1·6392
2·40E − 01	1·7416	1·7983
3·20E − 01	2·0254	2·0932
4·00E − 01	2·2916	2·3713
8·00E − 01	3·5602	3·6694
1·20E + 00	4·8172	5·0067
1·60E + 00	6·0738	6·3090
2·00E + 00	7·3304	7·6084
2·40E + 00	8·5870	8·9057
3·20E + 00	11·1002	11·4958
4·00E + 00	13·6136	14·0815
8·00E + 00	26·1800	26·9735
1·20E + 01	38·7464	39·8323
1·60E + 01	51·3126	52·6727

points where both the piezometric head and the flow rate are unknown. In this way the model comprises 6 boundary element regions and 229 nodal points, with 84 of them coinciding with the centroid of the finite elements which discretize the fractures. It is noted that regions 1,2 and 1,4 are such as to fulfill the continuity of flow condition orthogonally to the two fractures which are shown as dashed lines in the same Figure 6.

The unsteady-state flow analysis is intended to describe the flow regime during the first 13·12 days of injection. The initial condition assumes a zero piezometric head within the system. The results obtained are illustrated in Figure 7, where the hydraulic flow between the adjacent blocks and the fractures are depicted at 13·12 days. It is interest to point out the importance of the same fractures in determining the flow direction toward the zone of preferential drainage, as well shown by the equipotential lines plots given in Figure 8.

With the main purpose to illustrate the flow regime within the fracture system, figure 9 gives the piezometric heads at the following time steps: 1·312, 6·560 and 13·120 days. Also illustrated in

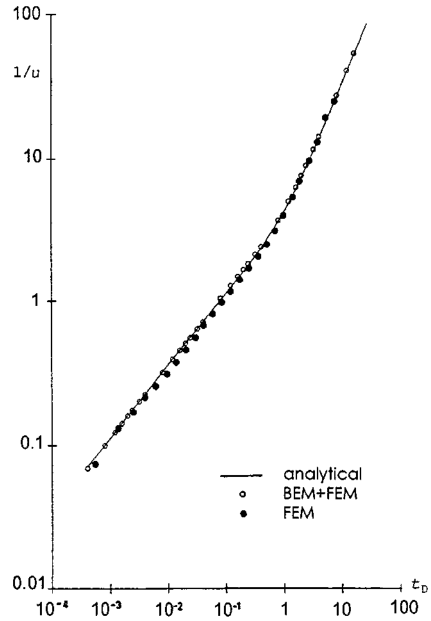

Figure 5. Comparison between analytical solution, FEM solution and the proposed FEM + BEM solution

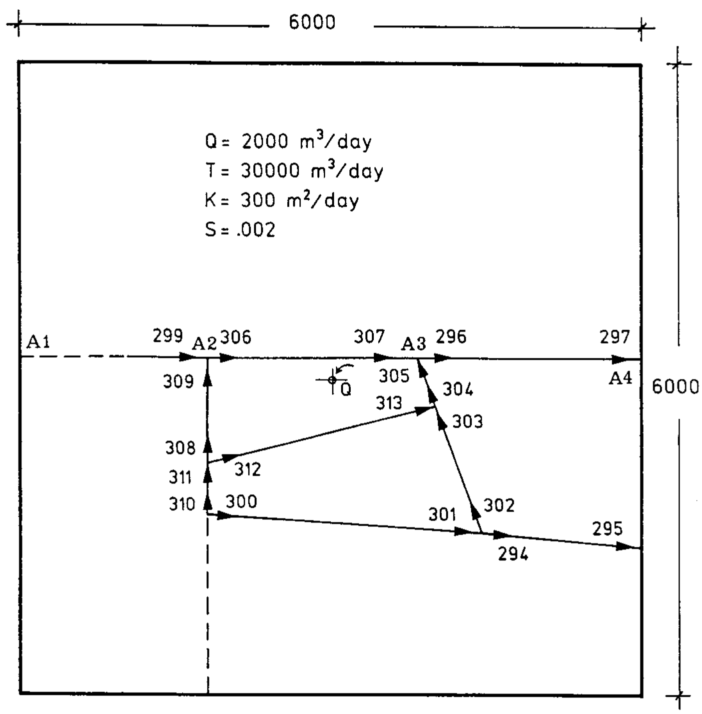

Figure 6. Scheme of the aquifer with the fracture net, its geometrical and hydraulic characteristics

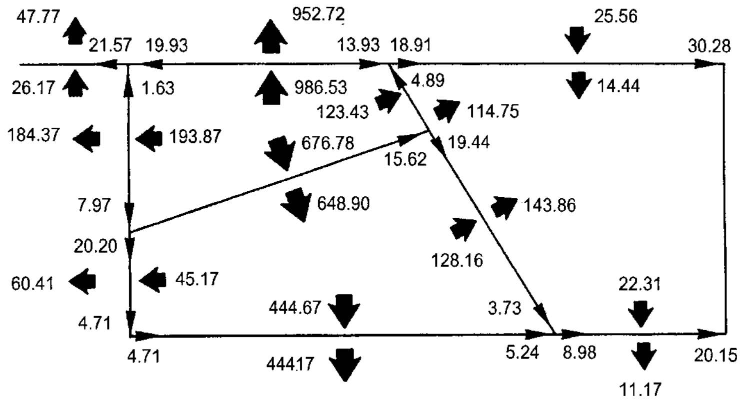

Figure 7. Values of the discharges along fractures and of the exchanged net flows ($Q_{\mathrm{net}} = \Sigma q_{\mathrm{n},l}$) between the blocks at 13·12 days

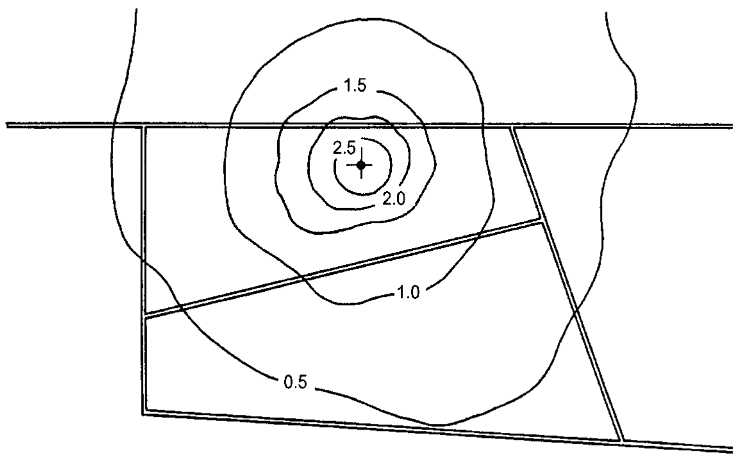

Figure 8. Hydraulic charges around the well at 13·12 days

figures 10 and 11, respectively, are the piezometric heads for flow occurring orthogonally to the A1–A4 fracture and the different stabilizing influence on flow due to the presence of the fracture facing block 3.

The case study shows that in order to determine the drainage properties of rock masses which are pervious and fractured it is essential to assess quantitatively the hydraulic interaction between the rock blocks and the discrete fractures. Therefore, the computational approach proposed here, which enables one to account for such a behavior, is to be considered as a valid technique to be preferred to a continuum equivalent approach.

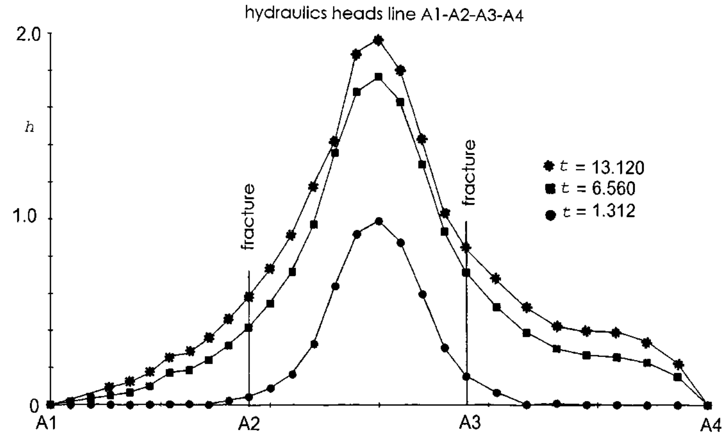

Figure 9. Hydraulic charges along A1-A4 at different time steps (t = 1·312, 6·560 and 13·120 days).

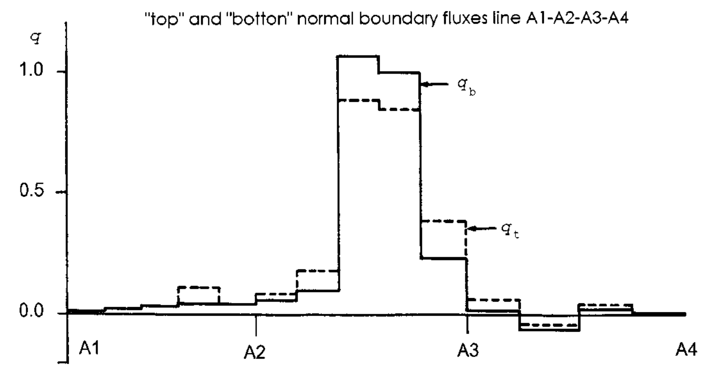

Figure 10. Exchanged normal boundary fluxes (top, bottom) between blocks along A1–A4 fracture at 13·12 days

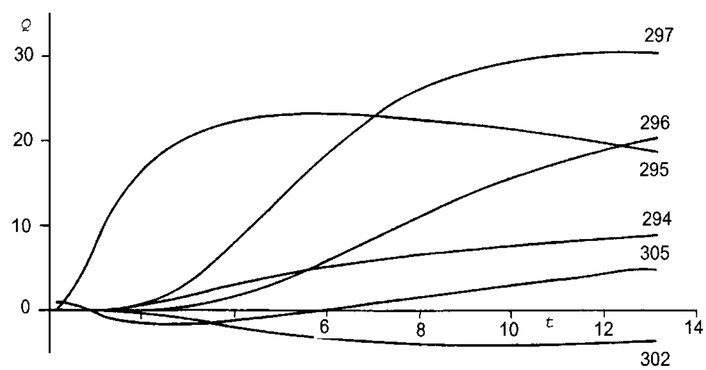

Figure 11. Discharges at fracture extremities in the time interval 0–13·12 days

5. CONCLUDING REMARKS

The present paper describes a hybrid of the finite element and boundary element method which is particularly useful for the solution of two-dimensional flow problems in unsteady-state conditions for fractured porous media. The pervious rock blocks are modelled implicitly using boundary elements. The discrete fractures are modelled using finite elements which allow for a better analysis of flow.

The distribution of the piezometric heads and of the flow between rock blocks and fractures, at various time steps, is obtained by solving the diffusion problem which can be written in terms of the continuity equation for the porous blocks, the discrete fractures, and the intersections of the same fractures, always accounting for the appropriate boundary conditions. The main features of such an approach are: the discretization, which is limited to the contours of the rock blocks only, and the possibility to proceed with time-steps chosen on the basis of the solution to be obtained.

The computational approach has been validated by comparing first the numerical solution with the corresponding closed-form solution, as described with case study 1. Then, a more complex problem has been analysed with case study 2, where the main features of the approach proposed have been used in modeling. While improving the computational efficiency of the whole model, the solution of a complex flow problem is obtained, where a discrete approach need be adopted as a better alternative to the equivalent continuum approach.

REFERENCES

1. P. S. Huyakorn and G. F. Pinder, *Computational Methods in Subsurface Flow*, Academic Press, New York, 1983.
2. C. R. Wilson, 'An investigation of laminar flow in fractured porous rocks', *Ph.D. Thesis*, Department of Civil Engineering, University of California, Berkeley, 1970.
3. A. M. Shapiro and J. Andersson, 'Steady-state fluid responses in fractured rock: a boundary element solution for a coupled, discrete fracture continuum model', *Water Resources Res.* **19** (4), 959–969 (1983).
4. T. W. Doe, J. D. Osnes, 'Interpretation of fracture geometry from well test', *Proc. Int. Symp. Fundaentals of Rock Joints*, Bjiorkliden, 1985, pp. 281–292.
5. J. Bear, *Hydraulics of groundwater*, McGraw-Hill, New York, 1979.
6. P. S. Huyakorn, B. K. Lester and C. R. Faust, 'Finite element techniques for modeling groundwater flow in fractured aquifers', *Water Resources Res.*, **19** (4), 1019–1035 (1983).
7. R. G. Baca, R. C. Arnett and D. W. Langford, 'Modeling fluid flow in fractured-porous rock masses by finite element techniques', *Int. j. numer. methods fluids*, **4,** 337–348 (1984).
8. Y. P. Chang, C. S. Kang and D. J. Chen, 'The use of fundamental Green's functions for the solution of problems of heat conduction in anisotrophic media', *Int. J. Heat Mass Transfer*, **16,** 1905–1918 (1973).
9. C. A. Brebbia, W. Telles and J. Wrobel, *Boundary Element Techniques; Theory and Applications in Engineering*, McGraw-Hill, New York, 1984.
10. A. C. Gringarten, H. J. Ramey and R. Raghavan, 'Unsteady-state pressure distributions created by a well with a single infinite conductivity vertical fracture', *Soc. Pet. Eng. J.*, **14,** 347–360 (1974).